Singular perturbation methods: Application in a hyperelastic compressible cylindrical tube

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Abstract: This study proposes a mathematical model to investigate stability of arteries. The artery is considered as a prestressed thick-walled tube subjected to dynamical pressure and made of a hyperelastic and composite material. In this context, the purpose of this work focuses on the initial formation of the pathology in human arteries which may be modeled as instability phenomena. For that, a perturbation technique is used on the equations of motion to highlight possible instabilities of the artery. This instability interpretation provides a theoretical approach.

Keywords: System of nonlinear equation, dynamic, hyperelastic,compressibility, exact solution, perturbation, stability.

1. Introduction

The study of the stability of an equilibrium point of a nonlinear ordinary differential equation is relatively trivial if the function that defines the equation is sufficiently regular one hand, and if the linearization at the equilibrium is hyperbolic.
In this case, we know that the differential equation is equivalent to the linearized equation [1].
But if the situation is not hyperbolic, the determination of stability becomes more difficult.
It then proceeds generally to the method of linearisation. [2].
We study the phenomena of instability by a perturbation technique.
The study of the existence and stabilization of the solution of the perturbed system is based on inequalities of integrals [3, 4], on systems Petrovsky [5] or on Lyapunov function.
The study of buckling of cylindrical shells under combined load of axial compression and pressure extrenal was made by Shen and Chen [6].
As for the analysis of the instability of cylindrical shells under hydrostatic pressure, we can cite the work of Barush and Singer [7].
The qualitative theory of differential equations in a self-analysis around a fixed point leads to consideration of a Stability around this point.
This leads naturally to a study of how the behavior near this point can change by changing the control parameters such as the Reynolds number in a hydrodynamic system. Such studies are now well developed [8], and methods varied.

The study of phenomena of stability in this work will be based on the assumption that the artery is similar to a cylindrical orthotropic [9].

The arterial wall consists of an elastic material that can undergo large deformations and large displacements.
The material of the wall is supposed transverse and anisotropic. The energy of such materials in a hyperelastic has been given by Spencer [10], and specific forms of these potentials have been studied [11, 12].
In this work, we study a part of the dynamic behavior of the shell subjected to pressure on the inside and outside and other conditions of stability.
The objective is to simulate conditions frequently encountered such as the development of aneurysms: an unstable behavior leads to a non-symmetrical shape of the vessel. Austin et al. [13] and Akkas [14] believe that instability due to the development of the aneurysm can result in behavior of the branching structure. For Simkins [15] and Hung [16], the aneurysm can become dynamically unstable due to pulsatile blood flow. Both studies are based on linear elasticity, despite the nonlinear behavior of hyperelastic structures and large deformations arising.
Cons by Shah and Humphrey [17] showed that a lesion such as aneurysm, can be dynamically stable if one takes into account the effects of solid-fluid coupling.

In this work, we use the technique called interference. The motivation of this work is a contribution to the study of asymptotic behavior, when the supposed positive control parameter tends to zero, the solution of an evolution problem with non-classical boundary conditions are unusual.

We study these phenomena by the instability behavior of a cylindrical shell, subjected to pressure on its inner and outer. The objective is to simulate conditions frequently encountered, such as the development of an aneurysm.

2. Basic Formulae

We consider that the artery is modeled by a hollow tube of circular cross section subjected to an intraluminal inflation, reflecting the loading pressure during a cardiac cycle.

We call configuration 0, the zero stress state at zero deformation; configuration 1, the prestressed condition caused by the presence of residual stress without any external change; and configuration 2, the state that reflects the deformed configuration of the vessel induced by the change in pressure.

The coordinate system chosen is that of the cylindrical configuration. M^R is a material point identified by its coordinates (R,Θ,Z) in the configuration 0, becomes the point M^ρ of coordinates (ρ,ϕ,ξ) in configuration 1, to finally become point M^r of coordinates (r,θ,z) in configuration 2. Subsequently, we take the notation "i" and "o" to express the respective inner and outer boundary of the vessel wall.

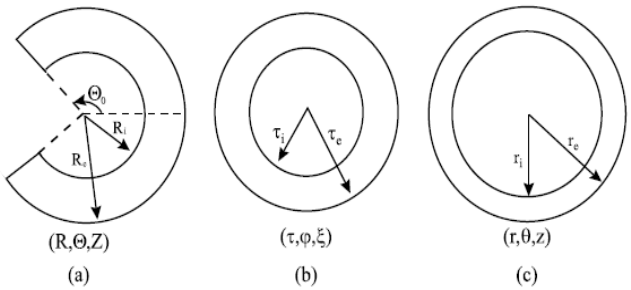

Fig.1:Cross-section of the tube in (a) stress-free, (b) unloaded, (c) loaded configuration

Based on the assumptions outlined above, one can describe the kinematics of the model. For this, we note $\mathbf{F}_1$ the gradient of the transformation that defines the transition from the configuration 0 to the configuration 1. This first deformation is described by:

$$\rho = \rho(R),\ \phi = \pi\frac{\Theta}{\Theta_0},\ \zeta = Z, (1)$$

where Θ_0 is the half-opening angle of the wall in the configuration 0.

Given (1), the matrix representation of $\mathbf{F}_1$ in the cylindrical base is written:

$$\mathbf{F}_1 = \begin{pmatrix} \frac{d\rho}{dR} & 0 & 0 \\ 0 & \alpha_0.\frac{\rho}{R} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad (2)$$

On the other hand, the configuration 2, which reflects the charged state is defined from a second gradient transformation $\mathbf{F}_2$ This second deformation is described by:

$$r = r(\rho,t) = r(R,t), \quad \theta = \phi,\ z = \lambda_0\xi. \ (3)$$

where λ_0 is the axial elongation and $\alpha_0 = \pi/\Theta_0$..

Given (3), the matrix representation of $\mathbf{F}_2$ in the cylindrical base is written:

$$\mathbf{F}_2 = \begin{pmatrix} \frac{\partial\rho}{\partial R} & 0 & 0 \\ 0 & \frac{r}{\rho} & 0 \\ 0 & 0 & \lambda_0 \end{pmatrix}. \qquad (4)$$

The gradient tensor of the total transformation $\mathbf{F}$ (configuration 0 to configuration 2) is thus:

$$\mathbf{F} = \mathbf{F}_2\mathbf{F}_1 = \begin{pmatrix} \frac{\partial r(R,t)}{\partial R} & 0 & 0 \\ 0 & \alpha_0.\frac{r(R,t)}{R} & 0 \\ 0 & 0 & \lambda_0 \end{pmatrix}. (5)$$

This allows expressing the strain tensor left Cauchy-Green $\mathbf{C}$:

$$\mathbf{C}=\begin{pmatrix}\left(\frac{\partial r(R,t)}{\partial R}\right)^2 & 0 & 0\\ 0 & \left(\alpha_0.\frac{r(R,t)}{R}\right)^2 & 0\\ 0 & 0 & \lambda_0^2\end{pmatrix}. \quad (6)$$

Note that for this kinematics of deformation, the strain tensor right Cauchy-Green is such that $\mathbf{B}=\mathbf{C}$.

If we consider a transverse isotropic material, and if $\mathbf{T}=[T_R,T_\Theta,T_Z]$ is the principal direction of orthotropy in the undeformed configuration, the isotropy of the material in the orthogonal directions to $\mathbf{T}$ implies that the hyperelastic potential W is expressed using the three invariants (I_1,I_2,I_3) of the Cauchy-Green tensor [18] and two pseudo invariants (I_4,I_5) [10]

$$\begin{aligned} &I_1=Tr(\mathbf{C}),\ I_2=Tr(Cof(\mathbf{C})),\\ &I_3=\det(\mathbf{C}),\\ &I_4=\mathbf{T}.(\mathbf{C}\mathbf{T}),\ I_5=\mathbf{T}.(\mathbf{C}^2\mathbf{T}). \end{aligned} \quad (7)$$

This potential $W(I_1,I_2,I_3,I_4,I_5)$ can be decomposed into two terms, an isotropic component W_{iso} and a component orthotropic W_{ort} [10].

If one considers a material of St Venant Kirchhoff, we obtain [18, 19].

$$\begin{aligned} W_{iso}&=\mu(I_1-3)+\frac{\lambda+2\mu}{8}(I_1-3)^2-\frac{\mu}{2}(I_2-3)\\ W_{ort}&=[\alpha+\beta(I_1-3)+\gamma(I_4-1)](I_4-1)-\frac{\alpha}{2}(I_5-1), \end{aligned} \quad (8)$$

where λ and μ are the Lame's constants, and

$$\begin{aligned} &\alpha=\mu-G_T;\ \beta=\frac{E\nu^2(1-n)}{4m_0(1+\nu)};\\ &\gamma=\frac{E_T(1-\nu)}{8m_0}-\frac{\lambda+2\mu}{8}+\frac{\alpha}{2}-\beta, \end{aligned} \quad (9)$$

with $m_0=1-\nu-2n\nu^2$ and $n=E_T/E$.

E_T and G_T denote the tensile modulus and shear in the direction of fibers.

With de neo-Hookeen material, the isotropic component is based on $J=\det(\mathbf{F})=\sqrt{I_3}$ by:

$$\begin{aligned} W_{iso}&=\mu(I_1-3)-\mu\ln(J)\\ &+(1/2)\ln(J-1)^2. \end{aligned} \quad (10)$$

The transverse component is written (6)

$$W_{ort}=\begin{bmatrix}\alpha+\beta\ln(J)+\\ \gamma(I_4-1)\end{bmatrix}.(I_4-1) \quad (11)$$
$$-\frac{\alpha}{2}(I_5-1).$$

Afterwards it is proposed to study the phenomena of instability in the behavior of the arterial wall. A solution of the equation of motion of a thick orthotropic shell is desired, and then a perturbation technique is used to highlight possible instabilities.

3. Equation of motion and radial solution

Define a constitutive law describing the mechanical behavior of hyperelastic nonlinear back to defining a constitutive relation between stress and strain. Recall that for a hyperelastic material, the Cauchy stress tensor $\boldsymbol{\sigma}$, is derivable from the strain energy function per unit mass [20, 21]. Specifically, it is represented as

$$\boldsymbol{\sigma}=\rho\mathbf{F}\left(\frac{\partial W}{\partial \mathbf{F}}\right)^{\mathrm{T}}, \quad (12)$$

where ρ is the density of the material in its deformed state, which is related to the density its reference configuration, $\rho_0=\rho\det(\mathbf{F})$.

The first stress tensor of Piola-Kirchoff $\boldsymbol{\pi}$ and the Cauchy stress tensor are given by

$$\boldsymbol{\sigma}=\frac{\boldsymbol{\pi}\mathbf{F}^{\mathrm{T}}}{J}=\frac{2}{J}\begin{bmatrix}W_1\mathbf{B}+(I_2W_2+I_3W_3)\mathbf{I}\\ -I_3W_2\mathbf{B}^{-1}+W_4((\mathbf{a}\otimes\mathbf{B}\mathbf{a}))\end{bmatrix}. \quad (13)$$

where $J=\det(\mathbf{F})$, $W_k=(\partial W/\partial I_k)\ (1\le k\le 5)$, $\mathbf{I}$ denotes the identity tensor and $\mathbf{a}=\mathbf{F}\mathbf{T}/\sqrt{I_4}$.

With the equations (10), (11) and (13), and in the absence of body forces, the equation of motion is given by:

$$\begin{aligned} &\frac{1}{\partial r(R,t)/\partial R}\frac{\partial\sigma_{11}(R,t)}{\partial R}+\\ &\frac{\sigma_{11}(R,t)-\sigma_{22}(R,t)}{r(R,t)}-\frac{\rho_0}{J}\frac{\partial^2 r(R,t)}{\partial t^2}=0. \end{aligned} \quad (14)$$

Working hypothesis, we choose an orientation of fibers as follows: $\mathbf{T} = [0,0,1]$, and taking into account the kinematics (3) and expressions of $W_k = (\partial W / \partial I_k)\ (1 \le k \le 5)$, the equation (14) becomes

$$\left[\begin{array}{l}\left(\alpha_0^2.\lambda_0^2.r(R,t)^3 + \mu.R^2.r(R,t)\right).\left(\frac{\partial r(R,t)}{\partial R}\right)^2 + \\ \beta.R^4.\left(1-\lambda_0^2+\mu\right)r(R,t)\end{array}\right].\frac{\partial^2 r(R,t)}{\partial R^2} +$$

$$\alpha_0^2.\lambda_0^2.r(R,t)^2.\left(\frac{\partial r(R,t)}{\partial R}\right)^4 +$$

$$2.\mu.R.r(R,t).\left(\frac{\partial r(R,t)}{\partial R}\right)^3 +$$

$$\left[\begin{array}{l}\alpha_0.\left(\mu.\alpha_0+2.\lambda_0.R\right)r(R,t)^2+\left(\mu+\beta-\lambda_0^2\right)R^4 \\ -\rho_0.R^2.r(R,t).\frac{\partial^2 r(R,t)}{\partial t^2}\end{array}\right].\left(\frac{\partial r(R,t)}{\partial R}\right)^2 +$$

$$8.R^2.\left(\beta.\lambda_0^2.R-\beta.R-\mu\right)r(R,t).\frac{\partial r(R,t)}{\partial R}$$

$$+12.\left(\beta.\lambda_0^2-\mu-\beta\right)R^2.r(R,t)^2 = 0. \qquad (15)$$

To study the phenomena of instability in the behavior of the arterial wall, we seek fist a trivial solution of equation (15) as:

$$r(R,t) = G(R).H(t). \qquad (16)$$

We limit the study of stability in a cardiac cycle that lasts about on second. This allows us to move to a limited development in the variable t and for convenience, we choose the first order.
The form

$$r(R,t) = R.(1+t), \qquad (17)$$

is a solution of the equation (15) if and only if

$$\begin{cases}\left(\alpha_0+\beta\right).\lambda_0^2-\alpha_0\lambda_0-\beta=0 \\ 4\gamma\alpha_0\lambda_0^5+\left(-4\gamma\alpha_0+\alpha_0\beta+\alpha_0^3+2\alpha_0\beta\ln(\alpha_0\lambda_0)\right)\lambda_0^3, \\ -\alpha_0^2\lambda_0^2+\left(\mu\alpha_0^3-\alpha_0\beta-\mu\alpha_0\right)\lambda_0=0\end{cases}$$

4. Linearization of the equations near the trivial solution

What happens if we perturb the solution (17) slightly?
To find the evolution of the fluctuations around this solution, we apply the technique standard linearization around this position. We put

$$r = r(R,t) = R.(1+t) + \varepsilon.\Psi(R,t), \qquad (19)$$

where ε is a small parameter quantifying the magnitude of disturbance and $\Psi(R,t$ to determine an unknown.

We set as a new kinematics [22]:

$$r = r(R,t) = R.(1+t) + \varepsilon.\Psi(R,t) = \varphi_0 + \varepsilon.\Psi,$$
$$\theta = \alpha_0\Theta,\ z = \lambda_0 Z,$$

where φ_0 is the solution defined in (17).
In the cylindrical base $(\vec{e}_r, \vec{e}_\theta, \vec{e}_z)$, $r = \varphi_0\vec{e}_r + \varepsilon.\Psi\vec{e}_r$.
We set $\mathbf{F}_0 = \mathrm{diag}(1+t, \alpha_0(1+t), \lambda_0)$, obtained by asking $r(R,t) = \varphi_0$ in (5).
We then write also:

$$\mathbf{F} = \nabla\left(\varphi_0 + \varepsilon.\Psi\right) = \mathbf{F}_0 + \varepsilon.\nabla\Psi = \mathbf{F}_0 + \varepsilon.\mathbf{\Phi}. \qquad (21)$$

The first tensor of Piola-Kirchhof is developed using the derivation in the sense of Gâteaux:

$$\boldsymbol{\pi} = \boldsymbol{\pi}_0$$
$$+2.\varepsilon\left\{\begin{array}{l}W_1^0\mathbf{\Phi}+W_2^0\left[\mathbf{\Phi}.\left(tr(\mathbf{C}_0)I-\mathbf{C}_0\right)+\mathbf{F}_0\left(\begin{array}{l}2\left(\mathbf{F}_0:\mathbf{\Phi}\right)I \\ -\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_0-\mathbf{F}_0^{\mathrm{T}}\mathbf{\Phi}\end{array}\right)\right]+ \\ W_3^0\left(\left(\det\left(\mathbf{F}_0\right)\right)^2\mathbf{F}_0^{-\mathrm{T}}\mathbf{\Phi}\mathbf{F}_0^{-\mathrm{T}}\right)+W_4^0\mathbf{\Phi}\left(\mathbf{a}\otimes\mathbf{a}\right)+ \\ +W_5^0\left[\begin{array}{l}\mathbf{F}_0\left(\mathbf{F}_0^{\mathrm{T}}\mathbf{\Phi}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_0\right)\left(\mathbf{a}\otimes\mathbf{a}\right)+\mathbf{\Phi}\left(\mathbf{C}_0\mathbf{a}\otimes\mathbf{a}\right) \\ +\mathbf{F}_0\left(\mathbf{a}\otimes\left(\mathbf{F}_0^{\mathrm{T}}\mathbf{\Phi}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_0\right)\mathbf{a}\right)+\mathbf{\Phi}\mathbf{a}\otimes\mathbf{C}_0\mathbf{a}\end{array}\right] \\ +\left[\frac{\partial W_1}{\partial\mathbf{F}}\left(\mathbf{F}_0\right):\mathbf{\Phi}\right]\mathbf{\Phi}+\left[\frac{\partial W_2}{\partial\mathbf{F}}\left(\mathbf{F}_0\right):\mathbf{\Phi}\right]\mathbf{F}_0\left(tr(\mathbf{C}_0)I-\mathbf{C}_0\right) \\ +\left[\frac{\partial W_3}{\partial\mathbf{F}}\left(\mathbf{F}_0\right):\mathbf{\Phi}\right]\left(\det\left(\mathbf{F}_0\right)\right)^2\mathbf{F}_0^{-\mathrm{T}} \\ +\left[\frac{\partial W_4}{\partial\mathbf{F}}\left(\mathbf{F}_0\right):\mathbf{\Phi}\right]\mathbf{F}_0\left(\mathbf{a}\otimes\mathbf{a}\right)+ \\ \left[\frac{\partial W_5}{\partial\mathbf{F}}\left(\mathbf{F}_0\right):\mathbf{\Phi}\right]\left[\mathbf{F}_0\left(\mathbf{C}_0\mathbf{a}\otimes\mathbf{a}\right)+\mathbf{F}_0\left(\mathbf{a}\otimes\mathbf{C}_0\mathbf{a}\right)\right]\end{array}\right\} \\ +O\left(\varepsilon^2\right), \qquad (18)$$

With $W_k^0 = W_k\left(\mathbf{F}_0\right)$ and $\boldsymbol{\pi}_0 = \boldsymbol{\pi}\left(\mathbf{F}_0\right)$ the tensor associated with $\mathbf{F}_0$.
The linearized problem is equivalent to:

$$\begin{cases} \text{Find } \Psi \in H^2(\Omega) \text{solution of} \\ \boldsymbol{\Phi} = \nabla\Psi, \\ \dfrac{1}{\partial r(R,t)/\partial R}\dfrac{\partial\sigma_{11}(R,t)}{\partial R} + \dfrac{\sigma_{11}(R,t)-\sigma_{22}(R,t)}{r(R,t)}, \\ -\dfrac{\rho_0}{J}\dfrac{\partial^2 r(R,t)}{\partial t^2} = 0 \\ r = r(R,t) = R.(1+t) + \varepsilon.\Psi(R,t) = \varphi_0 + \varepsilon.\Psi \end{cases}$$

where Ω is the field inside the cylinder.
In the equation of motion defined en (21), let

$$H(\Psi,\varepsilon) = \frac{1}{\partial r(R,t)/\partial R}\frac{\partial\sigma_{11}(R,t)}{\partial R} + \frac{\sigma_{11}(R,t)-\sigma_{22}(R,t)}{r(R,t)} - \frac{\rho_0}{J}\frac{\partial^2 r(R,t)}{\partial t^2}$$

and conduct limited development of the function $H(\Psi,\varepsilon)$ with order 2, under ε.

To resolve the problem (23), then we set:
$\Psi(R,t) = R + U(R).\cos(w.t)$
The equation of motion reduces to

$$\frac{\mathrm{d}^2U(R)}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}U(R)}{\mathrm{d}R} - \left[\begin{array}{l} \dfrac{\rho_0.w^2.}{\beta.(\lambda_0^2-1)-3\mu-\alpha_0^2\lambda_0^2} \\ +\left(\dfrac{\beta.(\lambda_0^2-1)+\mu-\alpha_0^2\lambda_0^2}{\beta.(\lambda_0^2-1)-3\mu-\alpha_0^2\lambda_0^2}\right).\dfrac{1}{R} \end{array}\right].U(R) = 0, \quad (24)$$

The boundary value problem amounts to solving the equation (24) with the following boundary conditions:

$$\begin{aligned} r(R_i,0) &= R_i \\ r(R_O,t) &= \tilde{r}_O(t), \end{aligned} \quad (25)$$

where $\tilde{r}_O(t)$ is the outer radius of the arterial wall measured at time t of a patient, R_i and R_O the inner and outer radii in the reference state. It should be noted that the shape of a $r(R,t)$ takes into account (19), (20) and form thing for $\Psi(R,t)$. These boundary conditions (25) correspond to the experimental data of kinematic variation of the arterial wall during a cardiac cycle in patients. Note that they were fitted by Fourier series as [23]

$$\begin{aligned} \tilde{r}_O(t) &= A_0 + \sum_{n=1}^{5}(B_n\cos(nwt) + C_n\sin(nwt)) \\ \tilde{r}_i(t) &= \tilde{A}_0 + \sum_{n=1}^{5}(\tilde{B}_n\cos(nwt) + \tilde{C}_n\sin(nwt)), \end{aligned} \quad (26)$$

Equation (24) has the form of a Bessel equation. Indeed, it is of the form:

$$\frac{\mathrm{d}^2U(R)}{\mathrm{d}R^2} + \frac{1}{R}\frac{\mathrm{d}U(R)}{\mathrm{d}R} - \left[m^2 + \frac{\vartheta^2}{R}\right].U(R) = 0, \quad (27)$$

with $m^2 = \dfrac{\rho_0.w^2.}{\beta.(\lambda_0^2-1)-3\mu-\alpha_0^2\lambda_0^2}$ and $\vartheta^2 = \dfrac{\beta.(\lambda_0^2-1)+\mu-\alpha_0^2\lambda_0^2}{\beta.(\lambda_0^2-1)-3\mu-\alpha_0^2\lambda_0^2}$.

Equation (27) thus admits as solution

$$U(R) = \bar{A}_0.I_\vartheta(m.R) + \bar{A}_1K_\vartheta(m.R), \quad (28)$$

With $\bar{A}_0$ and $\bar{A}_1$ integration constants obtained from the boundary conditions (25), $I_\vartheta(m.R)$ and $K_\vartheta(m.R)$ are modified Bessel functions of order υ respectively first and second kind. This leads to the solution of problem (23) and therefore the general shape of the radial displacement defined in (20):

$$\begin{aligned} r(R,t) &= R(1+t) + \varepsilon.\Psi(R,t) \\ &= R(1+t) \\ &+ \varepsilon.\left[R + \left(\begin{array}{l}\bar{A}_0.I_\vartheta(m.R) + \\ \bar{A}_1.K_\vartheta(m.R)\end{array}\right).\cos(w.t)\right]. \end{aligned} \quad (29)$$

5. Results and discussion

The model that we used took into account the residual stresses. Some authors have highlighted the presence of residual stresses in the arterial wall matrix. These constraints can be described by a nonzero opening angle when the arterial segment is dissected along its length. We based on the work of Anissa Eddhahak-Uni et al. [24] who proposed a range of value of angle $\Theta_0(°) \in [60,180]$. For the simulation we choose an angle of $120°$.

The determination of geometric parameters of the wall studied in the reference condition is very complex. To estimate the inner and outer radii in the reference state, we started observations of Delfino et al. [25].This allows us to consider $R_i = 3.21mm, R_O = 3.71mm.$

Figure 2 shows the influence of disturbances on the variation with time of the radial deformation at the inner surface of the arterial wall.
We note that the radial deformation increases with the disturbance. Inside the wall, and the theoretical point of view, we can assume that these disturbances may represent an aneurysm. Inflammation is also defined as the response of tissue to injury. Beginning as a small dilatation of the arterial wall, the disturbances can expand to over 10 cm in diameter as shown by Vorp [26]. This expansion may lead to a rupture. This confirms the work of Jiu-sheng REN et al. [27] who argue that the rupture of aneurysms is often followed by the formation of an intraluminal or intramural thrombus.

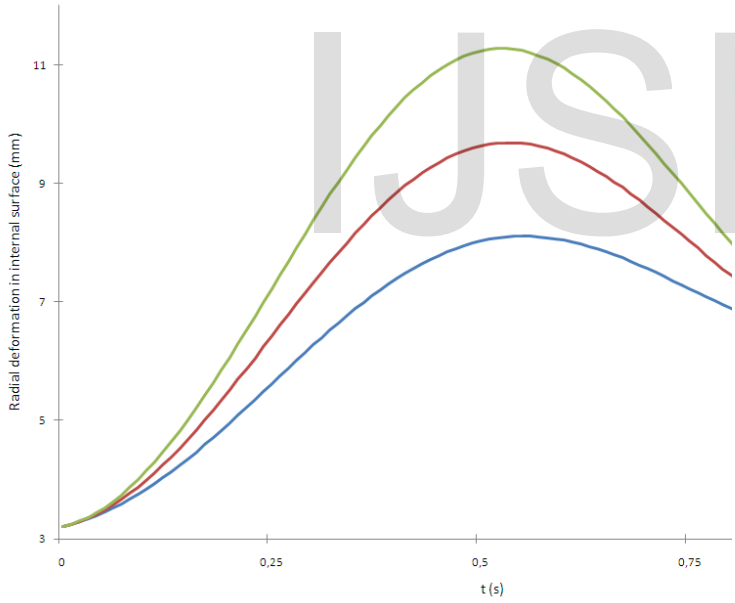

Fig. 2: Influence of perturbation on the radial deformation for $R = R_i$.

Figures $3a, 3b$ and $3c$, respectively show the influence of distu rbances on the stress distribution $\sigma_{rr}, \sigma_{\theta\theta}$ and σ_{zz} to R_i during a cardiac cycle.

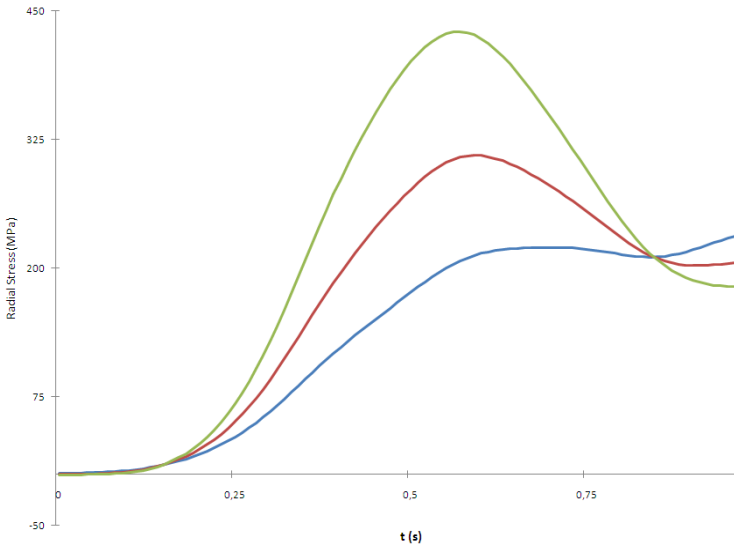

Fig. 3a: Influence of perturbation on $\sigma_{rr}(R = R_i, t)$

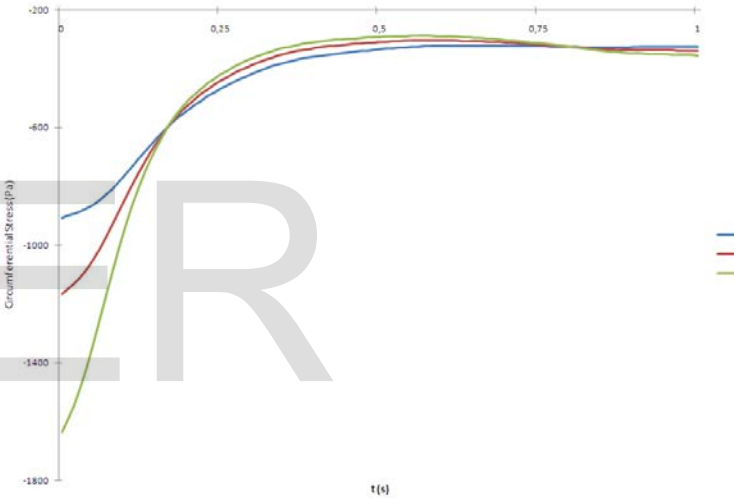

Fig. 3b: Influence of perturbation on $\sigma_{\theta\theta}(R = R_i, t)$

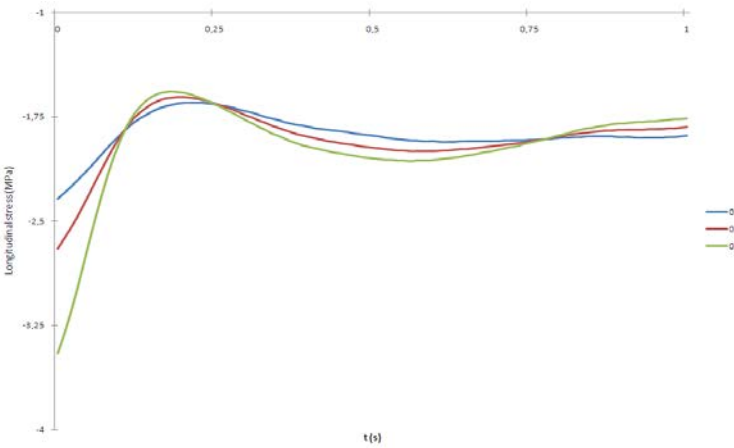

Fig. 3c: Influence of perturbation on $\sigma_{zz}(R = R_i, t)$

We find that the stress distribution mainly radial stress is very sensitive to disturbance. This is understandable given first described in the kinematic equations (3) and (19).

In interpreting these results in biomechanics and assimilate disruption ε to an aneurysm, we can argue that the larger the size of the aneurysm is more important radial stress is large. This will result in an inflammation of the structure, and therefore a risk of rupture. This confirms the work of Jiu-Sheng and Xue-gang [27]. The influence of disturbances that we note at the radial stress leaves appears instability phenomena in the arterial wall. In general, we say that a solution to a problem that it is stable if it is insensitive to variations in data.

It is important to note that the analysis of instability depends very much on the constitutive material model the artery wall. This model involves the derivatives (as John) potential relative to the first five invariants. we analyzed the dynamic behavior of a hyperelastic modeling the structure of an artery wall. Possibilities of instabilities have been implemented. Considering the disturbances as pathologies such as aneurysms, we believe that

the formation, enlargement, and rupture of an arterial wall can be described by the model presented in this paper and the effect of other factors is discussed.

Aneurysms can form in the arterial wall under abnormal conditions if the rigidity of collagen fibers decreased to some extent.

Aneurysms can be instability of the arterial wall,

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